The Kepler Problem

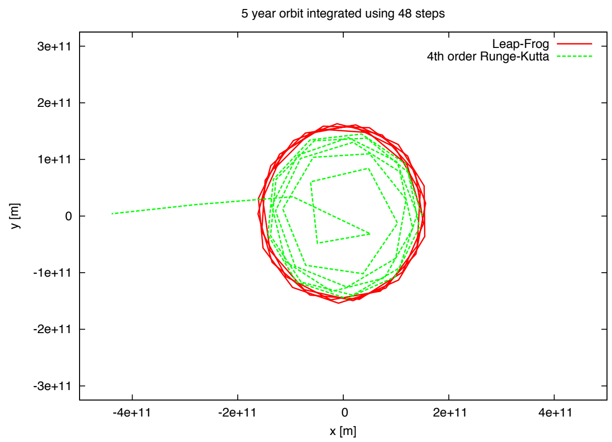
- we aim at comparing the leap-frog integration scheme to the 4th order Runge-Kutta scheme for the orbit of the Earth about the Sun.
- the relevant equations-of-motion for the Earth are

$$\frac{d\vec{r}}{dt} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = G\frac{M_{sun}}{|\vec{r}|^3}\vec{r}$$

where the Sun has been placed at the centre of the coordinate system and remains fixed there.

➢ **exercise:**

- write code integrating the equations-of-motion using both the leap-frog and the 4th order Runge-Kutta scheme.
- for each solution write a separate output file, e.g. leapfrog.dat & rk4.dat
- show the superiority of the leap-frog integrator when using, for instance, 48 integration steps for 5 full orbits of the Earth about the Sun:



The Kepler Problem

▪ **tips:**

• the whole problem is 2D as the orbit stays in a plane.

• you have to integrate the coupled set of four 1st order equations:

$$
\begin{aligned}
\frac{dx}{dt} &= v_x \\
\frac{dy}{dt} &= v_y \\
\frac{dv_x}{dt} &= -G\frac{M}{r^3}x \\
\frac{dv_y}{dt} &= -G\frac{M}{r^3}y
\end{aligned}
$$

• both integration schemes are described in

http://popia.ft.uam.es/aknebe/page3/files/ComputationalAstrophysics/03NumericsReview.pdf

• you might want to use the leap-frog scheme as given on p.132 as this assures temporal synchronisation with the Runge-Kutta scheme after each step; otherwise you should not forget the *jumpstart* at the very beginning of the integration and the *resync* when doing the very last step.